function to obtain velocities, strain rates, strains, and stresses at each grid point. The method is similar to that described earlier[7] though complications are caused by the non-uniform grid.[8]

Results and Discussion

Figs. 3 (*a*) and (*b*) show the flow lines in the midsection of the billet after extrusion under oblique and direct light, respectively. The first observation that can be made is that there was no sharp change of slope at contact between billet and die. The deformation of the billet before the die was reached is not pronounced but nevertheless was present and is more obvious from the velocity and strain fields.

It was also apparent that flow in the deformation did not follow either the radial or parallel flow paths normally assumed in upper-bound solutions. There was an initial region displaying a change of slope as the deformation zone was entered. Next there was a portion in which the line was nearly straight, then the slope of the flow line decreased slightly at a point roughly one-third of the way along the die. The slope then again increased and the line was once more approximately straight up to exit from the deformation region.

Fig 3(*b*) clearly shows the boundary where the polished billet had deformed sufficiently for its surface to become roughened. If it is assumed that this roughening took place at a constant value of strain, the outer fibres of the billet must have become deformed earlier than those near the central line, contrary to the assumption made in the upper-bound solutions of hydrostatic extrusions. 9, 10.

Attempts were made to estimate the deformation region by following the lines from the billet and product until they deviated from parallelism with the axis. It was very difficult to obtain reproducible results, especially near the centre where the change of curvature was least. When these data were used in the computer programme and smoothing was carried out between the deformation boundaries, the results near these boundaries were erratic because of the inaccuracy of the estimated boundary imposing a deformation pattern on the flow that had not occurred in practice. It was found advantageous to smooth the flow function between two radial sections—one in the billet and one in the product.

Throughout the strain-rate fields the influence of these effects can be seen. However, for comparing the frictional conditions operative during extrusion it is more convenient to use the velocity and total strain fields, as it is simpler to observe changes here (Fig. 4).

In all the extrusions the variation of total strain across the section was small, as would be expected in hydrostatic extrusion through a low die angle, the part of the product that is more heavily strain-hardened being confined to the region close to the surface. The difference in total strain between the central line and the surface was 20% max.—much less than the variations for the larger die angles used in conventional extrusion.[7]

The effect of extrusion speed on friction can be most clearly seen by comparing the axial velocity fields in two extrusions carried out under the same conditions but at different speeds, e.g. Nos. 4 and 5 in Table I and Fig. 4(*a*). At the higher speed, deformation occurred before the billet surface reached the die, whereas this effect was absent at low speed. Deformation before the die was reached can be explained by build-up of pressure in the molybdenum disulphide grease as it was dragged into the wedge formed by the billet and the die.

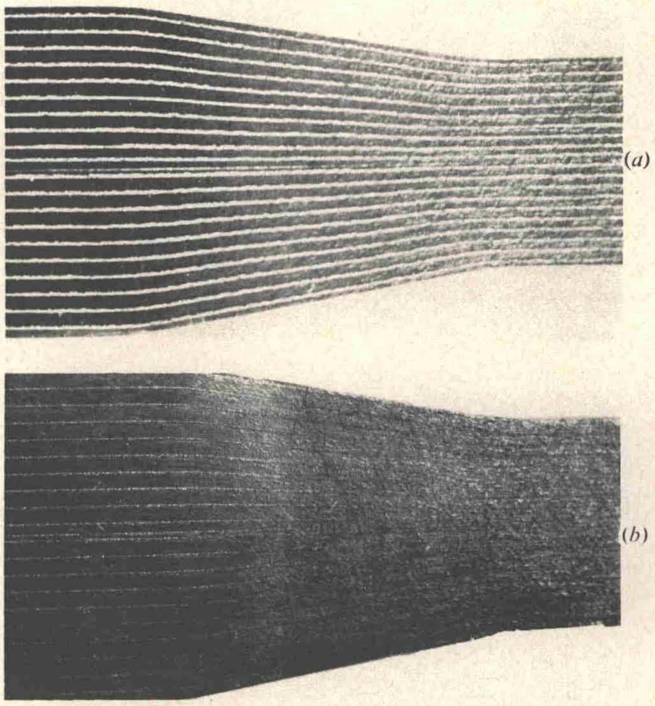

Fig. 3 *Flow lines: (a) Extrusion No. 3, under oblique light. (b) Extrusion No. 5, under direct light.*

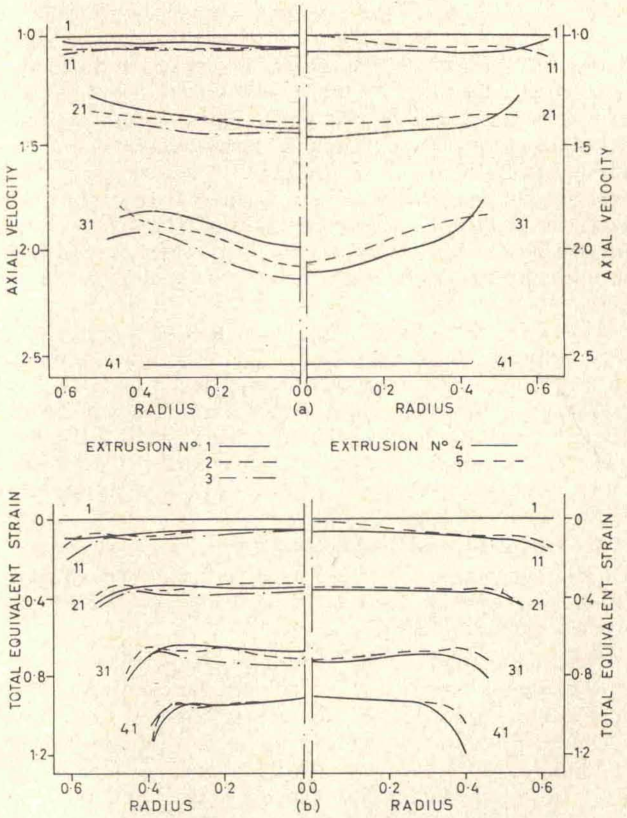

Fig. 4 *Axial velocity and total equivalent strain distribution in the deforming region: (a) Axial velocity; (b) total equivalent strain.*
Extrusion numbers refer to test numbers given in Table I. Numbers adjacent to the curves refer to axial co-ordinates in Fig. 2. Radius in cm.

TABLE I

Extrusion No.	1	2	3	4	5
Fluid pressure, N/mm^2	296·5	351	355·5	258	220
Drawing stress, N/mm^2	86·5	23·2	37·8	139·1	129·4
Temperature above ambient, degC	0°	3·5	1·0	4·0	5·3
Drawing speed, cm/min	9·31	9·27	9·05	8·89	82·5
Pressurizing fluid	Tellus 27	Castor oil	Castor oil	Castor oil	Castor oil
Lubricant	None	None	Molyslip	Molyslip	Molyslip
Ambient temperature, °C	20	19	27	27	21

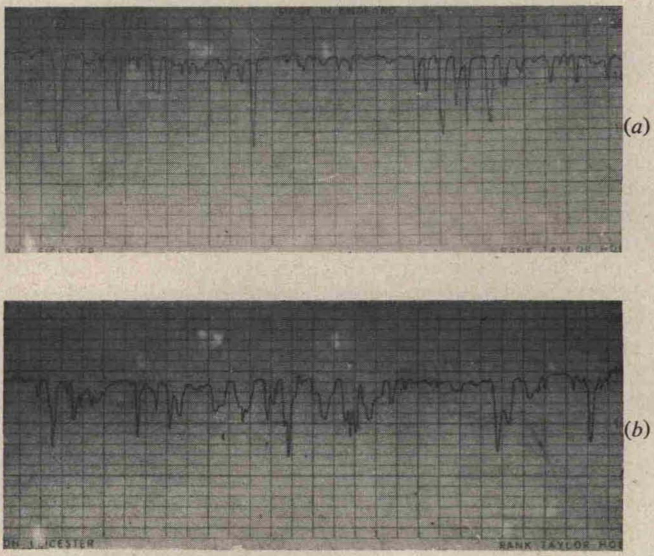

Fig. 5 Surface profile of the extruded product: (a) At 8·89 cm/min drawing speed; (b) at 82·5 cm/min drawing speed.

This phenomenon is known as the 'viscosity-pump' effect.[11] As pressure increase is directly proportional to speed, other factors being unchanged, the speed ratio of 9 for the two extrusions explains why the effect is present only at higher speeds.

It should be noted that Reynolds' equation for this flow situation on the assumption of total contact between billet and die (namely zero lubricant flow), results in an infinite pressure build-up. There must, in fact, be some flow of lubricant or fluid through the die; and also, when the pressure build-up is sufficient the billet will yield, the changed geometry relieving further build-up of pressure.

If this explanation is correct, lower friction is to be expected over the whole die face at higher speeds as more lubricant is forced to the interface. That this is indeed the case can be seen from the axial velocity and total strain fields. The variation of velocity and strain across the section is much smaller for the high-speed extrusion, even near the exit from the die. It is also interesting to note that axial velocities are higher near the billet/die interface than deeper inside the billet in the higher-speed extrusion 5, an effect that is quite pronounced as far as section 21 (Fig. 4(*a*)). The lubricant penetrating between the billet/die interface must be responsible for the much more uniform velocity distribution at higher speeds, as far as section 31, quite close to the die exit.

Direct evidence that the lubricant flow rate is greater at higher speeds can be gained from Fig. 5. This shows the surface profiles in the longitudinal direction for two extrusions and it can be seen that burnishing is much increased at lower speed. The greater lubricant flow at higher speed is therefore effective by forming pockets and preventing contact over a large area. The profiles of the two billets were checked and found to be very similar.

To understand the dependence of friction on extrusion-stress/drawing-stress ratio, we must consider two extrusions in which the other parameters were identical, i.e. Fig. 4, Table I, extrusions 3 and 4.

Here, again, deformation before the die is reached occurs in the extrusion-orientated case. When the drawing stress is high, the extrusion pressure is correspondingly lower, and therefore to cause an initial yield in advance of the die, the build-up of pressure by the hydrodynamic effect would have to be greater by (approximately), the amount of the drawing stress.

The velocities may again be compared to show the variation of friction over the die surface. With extrusion at lower drawing stress the build-up of pressure is sufficient to cause yielding. This reduces friction, as is evidenced by Fig. 4(*a*), where the velocity is shown to be more uniform for the extrusion-orientated case (extrusion 3).

Considering the total strain distribution, Fig. 4 (*b*), it can be seen that there is less variation of total strain across the product for low drawing stress (extrusion 3). Also, the mean values of total strain and the total strain along the centre line are lower for the extrusion-orientated case, which indicates that the redundant strain is then smaller. The smaller redundant strain must be attributable to reduced friction as other parameters were the same for the two extrusions.

Increase in fluid pressure produces higher axial velocities near the surface, an effect similar to that found at higher speeds, though less pronounced. This indicates that within the range of pressure and speed ratios investigated extrusion speed has a more pronounced effect on lubrication than has an increase in fluid pressure.

In the present work, for otherwise identical conditions, the driving stress (sum of extrusion pressure and drawing stress) remained constant for various extrusion-pressure/drawing-stress ratios, while another investigator[12] reported decreasing driving stress in the drawing-orientated process when, according to the present results, friction increases owing to less-efficient lubrication. The probable explanation of this apparent contradiction lies in the different stress/